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Thesis

Influence of Theory and Experiment on the Most Probable Value of Seven Principal Physical Constants

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Thesis

INFLUENCE OF THEORY AND EXPERIMENT ON THE MOST PROBABLE VALUE OF SEVEN PRINCIPAL PHYSICAL CONSTANTS

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Influence of Theory and Experiment on the Most Probable Value of Seven Principal Physical Constants

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In 1936, Eddington published a book entitled "Relativity Theory of Protons and Electrons"[8] which was the outcome of several years of research. One of the features of this work was the establishment of a theoretical relationship between seven of the most fundamental constants of nature. It is the purpose of this paper to utilize Eddington's equations and Raymond T. Birge's compilations of experimental values of the physical constants to get values which should be of an improved order of precision if Eddington's theory turns out to be correct.

As a test, Birge's 1929[1] table of constants (see Table I for excerpts) will first be employed. The recomputation of seven fundamental constants by the use of Eddington's theory may then be checked by the use of Birge's 1941 table of constants.[2] If the recomputation of the 1929 data turns out to be in the direction of the 1941 data, then a recomputation of the 1941 data will be made for the purpose of a check some time in the future.

The general method of procedure will be first to search in nature for natural units of length, time, and mass. In terms of the units which have been used in this paper, six of the seven constants under discussion may be evaluated to as many significant figures as we like.* These are

c, the speed of light = 1.0000000 natural units

λ, the cosmical constant of relativity theory = 1.0000000 natural units

G, the gravitational constant, = $\pi/2$ = 1.5707963 natural units

m_e, the rest mass of the electron = $3.4603443 \times 10^{-83}$ natural units

M_p, the rest mass of the proton = $6.3466635 \times 10^{-80}$ natural units

and h, Planck's constant = $6.7983239 \times 10^{-120}$ natural units

*This is true since they are functions of integers and π.

The other constant, e, may be written in terms of k, the dielectric constant of a vacuum which ironically enough may be expressed in c.g.s. units to as many figures as we like, but not so in the natural units.

$$e = 0.3869085 \times 10^{-62} k^{\frac{1}{2}}$$

One of these constants, λ, is the reciprocal of the square of the hyper-radius of the Einsteinian "finite but unbounded universe". The present theory makes this a function of Rydberg's constant together with a group of numbers which may be written down to any number of significant figures. Since we know Rydberg's constant to eight or nine significant figures we are in the slightly absurd predicament of having a theoretical value good to eight figures of a constant so little known experimentally that we merely guess at its order of magnitude.

So we shall take the "radius" of the universe as our natural unit of length; the time it would take light to travel this distance will be our natural unit of time although this radius is outside of the three-dimensional universe in which light actually does its traveling. The natural unit of mass will be the mass of the universe, since according to the theory the universe contains a known number of particles.

Having selected these natural units, it will be necessary to tie the arbitrary c.g.s. units to them by utilizing numerous independent determinations of natural constants. Let these relationships be expressed by the equations

U_L cm. = 1 natural unit of length = 1 nat.-cm.

U_T sec. = 1 natural unit of time = 1 nat.-sec.

U_M gm. = 1 natural unit of mass = 1 nat.-gm.

Therefore, these three quantities will serve as conversion factors, to

shift from the theoretical values of the constants which have high precision to c.g.s. values. The precision of the c.g.s. values of the constants will be limited by the precision of U_L, U_T, and U_M.

U_L cm./nat.-cm. is good to eight significant figures as we have seen. The precision of U_T sec/nat.-sec. is only as good as the value of the velocity of light, namely to six significant figures. U_M gm./nat.-gm. must be found as the weighted mean of numerous values.

Once these three conversion factors have been established, λ may be written $\frac{1}{U_L^2}$ cm.$^{-2}$

c becomes $U_L U_T^{-1}$ cm./ sec.

$G = 0.5\pi U_L^3 U_M^{-1} U_T^{-2}$ cm.^{3}/gm.-sec.^{2},

$m_e = 3.4603443 \times 10^{-83}\ U_M$ gm.

$M_p = 6.3466635 \times 10^{-80}\ U_M$ gm. and

$h = 6.7983239 \times 10^{-120}\ U_M\ U_L^2 U_T^{-1}$ gm.-cm.^{2}/sec.

k, the dielectric constant of empty space is 1.0000000 in c.g.s. units and $U_M\ U_L\ U_T^{-2}$ in natural units. Therefore

$$e = 8.8869035 \times 10^{-62}\ U_L\ U_M^{\frac{1}{2}}\ U_L^{\frac{1}{2}}\ U_T^{-1} \text{ c.g.s.e.s.u.}$$

Bearing these general features in mind it will perhaps not be too difficult to follow the details in what follows.

It will be observed that there still remain constants in Table I which could not be derived from theory, but which depend on experimental reconcilements of parallel systems of measurements. For example, such things as 1000.027 cm.^{3}/liter, the atomic weight of oxygen, the freezing point of water (273.18°K) all depend on purely arbitrary choices of man-made scales.

TABLE I.

Velocity of light	c =	$(2.99796 \pm 0.00004) \times 10^{10}$ cm/sec.
Gravitational constant	G =	$(6.664 \pm 0.002) \times 10^{-8}$ dyne-cm^2/gm^2
Liter	=	1000.027 ± 0.001 cm^3
Volume of perfect gas(0°, A[illegible])	R =	22.4146 ± 0.0008 liter/mole
Atomic weights	O =	16.0000 $H = 1.00777 \pm 0.00002$
Ice point (Absolute scale)	T_o =	$273.18 \pm 0.03°$ K
Mechanical equivalent of heat	J =	4.1852 ± 0.0006 abs. joule/cal.
Faraday constant	F =	96494 ± 5 int.-coul/gr.-equivalent
Electronic charge	e =	$(4.770 \pm 0.0005) \times 10^{-10}$ abs. e.s.units
Planck constant	h =	$(6.547 \pm 0.008) \times 10^{-27}$ erg.-sec.
Acceleration of gravity (45°)	g =	980.616 cm/sec^2
Rydberg constant for hydrogen	R_H =	109677.759 ± 0.05 cm.$^{-1}$
Rydberg constant for infinite mass	R_∞ =	$1[illegible]737.42 \pm 0.06$ cm.$^{-1}$
Avogadro's number	$N_o = F_c/e$ =	$(6.064[illegible] \pm [illegible].006) \times 10^{23}$ mole^{-1}
Mass of electron (spectroscopic)	$m_o = e/c(e/m)_{sp}$ =	$9.035[illegible] \pm 0.010) \times 10^{-28}$ g.
Mass of electron (deflection)	$m_o = e/c(e/m)_{defl}$ =	$([illegible].994[illegible] \pm 0.014) \times 10^{-28}$ g.
Mass of proton	$M_p = (H-m)/N_o$ =	$(1.6608[illegible] \pm 0.0017) \times 10^{-24}$ g.

At this point it seems proper and fitting to say something about the history of these constants. The speed of light has been determined experimentally by many physicists with very accurate results. Apparently Olaf Roemer (1644 - 1710) was the first to find a value for the speed of light. His value was 192000 miles per second. The following table is a list of the men with their determinations of the speed of light.

TABLE II.[6]
Leading Determinations of the Velocity of Light

Investigator	Year	x 10^{10} cm./sec.	Method
Roemer[6]	1675	3.08	Jupiter's satellites
Bradley[6]	1728	2.98	Aberration of light
Fizeau[4]	1849	3.15	Toothed wheel
Foucault[5]	1850	2.98	Rotating mirror
Cornu[7]	1875	2.99990 ± 0.00200	Rotating mirror
Michelson[10]	1880	2.99910 ± 0.00050	Rotating mirror
Newcomb[6]	1883	2.99860 ± 0.00030	Rotating mirror
Michelson[10]	1883	2.99853 ± 0.00060	Rotating mirror
Perrotin[6]	1902	2.99901 ± 0.00050	Toothed wheel
Rosa and Dorsey[6]	1906	2.99781 ± 0.00010	e.s.u/e.m.u.
Mercier[6]	1923	2.99782 ± 0.00030	Waves along wires
Michelson[11]	1926	2.99796 ± 0.00004	Rotating mirror
Mittelslaedt[6]	1928	2.99778 ± 0.00010	Kerr cells
Pease and Pearson[6]	1932	2.99774 ± 0.00002	Rotating mirror (vacuum pipe)
Anderson[6]	1937	2.99764 ± 0.00015	Kerr cells
Birge[2]	1941	2.99774 ± 0.00004	Weighted mean of the preceding

Three men have determined the value of the charge on the electron. R. A. Millikan's[3] investigations on this subject, by the oil-drop method extended over a period of many years. Wadlund[13] and Bachlin[12] utilized the absolute wave-lengths of X-ray lines. Finally, Birge[2] published the weighted mean of Wadlund's[13] and Millikan's work as his value of the charge on the electron.

Planck's constant "h" has been evaluated in a number of different ways.

One of these is a solving for "h" in the Bohr's formula for Rydberg's constant for infinite mass.

$$R_\infty = \frac{2\pi^2 e^5}{h^3 c^2 \frac{e}{m}}$$

The Rydberg constant for infinite mass is R_∞. R for any particular atom is found by replacing m_e, the mass of the electron, by μ where $\frac{1}{\mu} = \frac{1}{m_e} + \frac{1}{m_{atom}}$.

Other methods are: Ionization Potentials, Photoelectric effects, Stefan-Boltzmann Law and the Planck Equation, Wien's Displacemant Law and the Planck Equation, and finally the weighted mean of the results of all these methods as determined by Birge.[2]

P. R. Heyl,[9] according to Birge "has made what is undoubtedly the most reliable determination of the gravitational constant". Henning and Jaeger have also published their determination of G.

The mass of the electron and the mass of the proton are properly classed as derived constants and as such have been determined from the results of the work on the constants already spoken about. From these constants and with the help of Avogadro's number, N_o, and the Boltzmann constant k, most of the other physical constants may be derived. For example the Faraday constant F equals $\frac{eN_o}{c} \frac{\text{ab coulombs}}{\text{gm-equiv.}}$. The Stefan-Boltzmann constant $\sigma = \frac{2\pi^5 k^4}{15c^2h^3} \frac{\text{ergs}}{\text{cm}^2\text{deg}^4\text{sec.}}$. Wein's displacement constant is another derived constant $W = \frac{hc}{k4.9651}$ where 4.9651 is the root of the equation $e^{x} + \frac{x}{5} = 1$. Still another derived constant is the fine structure constant α. It is

derived from this equation $\alpha = \frac{2\pi e^2}{hc}$. For theoretical relationships we shall be interested in four equations developed by Eddington.[8]

(1) $\frac{hck}{2\pi e^2} = 137.00000$

(2) $\frac{M_p}{m_e} = 1834.1133 \ . \ . \ . \ .$

(3) $\frac{136}{137\pi} \quad \frac{\sqrt{10(136)2^{256}}}{\sqrt{3}} \qquad \frac{e^2}{C(1834.1133)m_e^{\ 2}k}$

(4) $\frac{1}{\sqrt{\lambda}} \qquad \frac{2G\left[136(2^{256})(1835.1133)m_e\right]}{\pi c^2}$

Explanation of Eddington's Equations.

It is well known that the modern reconciliation between the wave theory of light and the photon theory starts with the fact that all the energy of light is represented by the photons, and the waves must be recognized as so nearly devoid of energy as to take them quite out of the field of physics (which confines itself to the manifestation of energy). Two alternative points of view present themselves.

One possible point of view is that adopted in the current theory of wave mechanics, namely that the mathematical functions representing these light waves may be multiplied by their complex conjugate functions, to produce a product which serves as a probability function describing the chances of finding a photon in a given element of volume dx dy dz in a given time dt. In other words the waves are purely mathematical, sort of square roots of probability functions.

The other possible point of view is to think of the waves as similar to physical waves but as existing in a multidimensional space of which the

space of the physicist is a three-dimensional cross-section. This three dimensional cross-section may be visualized as a sort of hyper-surface which contains all the energy of the physical "universe". The radius of curvature of this hyper-sphere is Einstein's "radius", R, of his finite but limitless universe.

By dropping down a dimension or two we can get hints of what may be going on. If we had a one-dimensional universe embedded in a two-dimensional hyper-space, a wave in the latter would become evident to an observer in the former only as a series of moving points. The phenomena of interference and polarization would not carry over into the one-dimensional universe.

If the physical universe were two-dimensional but embedded in a three-dimensional hyper-space, we could carry out Young's interference experiment in the latter, say on something resembling the surface of a pool of mercury. If we let our two-dimensional physical observer be confined to a plane just above what would correspond to the equilibrium position of the "mercury surface", he would observe a succession of diamond-shaped figures gliding along his plane, due to the interference of the three-dimensional waves. In other words, the two-dimensional physicist would observe quanta which were "caused" by wave phenomena in three dimensions. There would be in this case no physical cause for the quanta since the three-dimensional waves would be extra-physical. In this illustration the effect of interference carries over into the behavior of the quanta, but not the effect known as polarization.

If the physicist be three-dimensional but imbedded in a still higher

dimensional extra-physical space, both interference and polarization effects can be carried over.

Minkowski fused three-dimensional space with one-dimensional time to produce a four-dimensional space-time manifold, but for our purpose, we require more dimensions than this.

Einstein put a curvature into Minkowski's four-dimensional space, making it Riemannian instead of Euclidean. However, Einstein's four-dimensional Riemannian space, which must still be considered as physical, may also be represented by ten Euclidean dimensions.

Eddington has shown that the ten dimensions are needed for physical phenomena but the ten dimensions must be considered as imbedded in a sixteen dimensional space of which six dimensions must be considered as non-physical. This, however, is a bare minimum, sufficing for point-masses only. If we wish to show relationships between two point-masses, we must increase the number of dimensions to 16 x 16 or 256. Of these, 6 x 6 + 10 x 10 or 136 are physical and 6 x 10 + 10 x 6 or 120 are extra-physical.

It takes a book of 329 pages for Eddington to explain this theory in an adequate fashion. Let it suffice to say here that we shall expect combinations of the integers 1, 2, 3, 4, 10, 136, 137, 256 along with π to appear in the results of his analysis.

For example we expect α, the fine structure constant to be exactly 1/137 on this theory.

The roots of the equation (See Eddington[8], p. 219)

$$10x^2 - 136x + 1 = 0$$

are

$$x_1 = 13.59264308$$

$$x_2 = 0.007356920903$$

and $$\frac{x_1}{x_2} = 1847.599459 \ . \ . \ .$$

$$\frac{136}{137} \text{ of } \frac{x_1}{x_2} \text{ or } 1834.113317 \ . \ . \ . \ . \ .$$

may be shown to be the expected ratio of the rest-mass of the proton to the rest-mass of the electron.

The constants in the other two equations on page 7 will likewise be recognized as made up of the same set of integers. Our problem is not to derive these equations but to investigate their validity in terms of experimentally determined physical constants.

In order to determine better values of these physical constants than is at present known, we shall follow the following scheme of procedure. We shall have as material to work with, the weighted mean of experimental values, as tabulated by Birge, and the theoretical relationships as formulated by Eddington. Now a word or two about weights and measures. If it seems convenient to have a different unit of length from one determined by act of Congress, that will not cause any dismay if we are able eventually to express our arbitrarily chosen unit of length in terms of centimeters. Or if it seems best to select a "natural" unit of weight such as the mass of an electron, that too will be all right just as long as we can express that "natural unit of mass" in terms of grams.

In order to derive the values of natural constants from theory, we need to anchor the gram, the centimeter, and the second to natural physical entities, but we must likewise tie together these units with such generally accepted (but purely arbitrary) systems of units as the international scale of atomic weights, the common temperature scales, etc. Given these relationships along with the values of seven fundamental constants, we may derive

all the rest. In other words it must be possible to change from one system of weights and measures to another. For example, the Avogadro constant, N_o, is a constant which depends on the relationship between the c.g.s. system and the atomic weight system. It could never be derived from pure theory.

$$N_o = (6.02283 \pm 0.0011) \times 10^{23} \text{ molecules per mole}$$

Likewise the equation

$$1 \text{ statom}^* = 1.6588599 \times 10^{-24} \text{ gms.}$$

depends on the relation between the same two systems. However, these two constants are related, as may be seen by the following argument. The mass of one hydrogen atom is the sum of the mass of the electron, m_e, and the mass of the proton, M_p.

$$M_p + m_e = 1.6709808 \times 10^{-24} + .0009111 \times 10^{-24} \text{ gm.}$$

$$\frac{1.6718919 \times 10^{-24} \text{ gm/H-atom}}{1.6588599 \times 10^{-24} \text{ gm/statom}} = 1.007856 \text{ statom/H-atom}$$

and $2(1.007856) = 2.015712$ statoms/H-molecule.

Now the mass of the hydrogen molecule, $H_2 = 3.3437838 \times 10^{-24}$ gm, and

$$N_o = (6.02283 \pm 0.0011) \times 10^{23} \text{ molecules/mole.}$$

Therefore $N_o \times H_2 = 2.0139043$ gm. = mass of 1 mole, which should coincide with 2.015712 statoms/H_2.

The Kelvin temperature scale also is arbitrary and may be linked to the c.g.s. system by the Boltzmann constant.

The units of measure that I have selected may seem remarkably imaginative at first glance. But if a little patience is observed I'm sure that before the mathematical reasoning has been completed the units will

* 1 "statom" (name suggested by G.F.Hull) $= \frac{1}{16}$ the weight of an oxygen atom.

not appear unreasonable, especially as they are expressed in units of the c.g.s. system.

As has been said, our unit of length is equal to the length of the Einstein "radius" of the universe. How long in actual centimeters is the "radius" of the universe? From Eddington's theory this "radius" is equal to the reciprocal of the square root of λ, the cosmical constant of relativity theory. We must, therefore, calculate the value of λ. It must be remembered that R_∞, the Rydberg constant for infinite mass, is the most accurately determined physical constant. If it is possible to show that λ is a function of R_∞ alone then, it follows that the value of λ, which experimentally is known merely as an extremely small quantity, would be capable of theoretical determination to an absurdly high degree of precision. It will now be demonstrated that λ is a function of R_∞ alone.

Consider the following equations, the first four due to Eddington and the fifth to Bohr:

(1) $$\frac{hck}{2\pi e^2} = 137$$

(2) $$\frac{M_p}{m_e} = 1834.1133\ldots,$$ a number which may be computed to any required degree of precision.

(3) $$\frac{136}{137\pi}\,\frac{\sqrt{10(136)2^{256}}}{\sqrt{3}} = \frac{e^2}{G(1834.1133 m_e{}^2 k)}$$

(4) $$\frac{1}{\sqrt{\lambda}} = \frac{2G\,136(2^{256})(1835.1133)m_e}{\pi c^2}$$

(5) $$R_\infty = \frac{2\pi^2 e^4 m_e}{ch^3}$$

From (4) $\lambda = \dfrac{\pi^2 c^4}{4G^2(136)^2 \times 2^{512}(1835.1133)^2 m_e^{\;2}}$

or $\lambda = \dfrac{c^4(2.9544619 \times 10^{-165})}{G^2\ m_e^{\;2}}$ (6)

From (1) $e^2 = \dfrac{hc}{860.79640}$ where k, the dielectric constant of a vacuum is taken as unity. (7)

From (3) $2.2893740 \times 10^{39} = \dfrac{e^2}{Gm_e(1834.1133)m_e}$ where k = 1 (8)

From (5) $R_\infty = \dfrac{19.739209 e^4 m_e}{c\ h^3}$ (9)

Substituting (7) in (9)

$$R_\infty = \frac{19.739209\cancel{h^2}c^{\cancel{4}}m_e}{(860.79640)^2\cancel{c}h^{\cancel{3}}} \qquad (10)$$

Substituting right side of (7) for e^2 in (8) and solving for c

$$c = 36.144594 \times 10^{44}\ \frac{Gm_e^{\;2}}{h} \qquad (11)$$

$$\lambda = 2.9544619 \times 10^{-165}\ \frac{c^4}{G^2 m_e^{\;2}} \qquad (12)$$

Substituting (11) in (10)

$$R_\infty = \frac{19.739209(36.144594 \times 10^{44})Gm_e^{\;3}}{(860.79640)^2\ h^2} \qquad (13)$$

Substituting (11) in (12)

$$\lambda = \frac{(36.144595 \times 10^{44}\ Gm_e^{\;2})^4 \times 2.9544619 \times 10^{-165}}{h^4\ G^2\ m_e^{\;2}} \qquad (14)$$

$$\lambda = \frac{(36.144594 \times 10^{44})^4\ G^{\overset{2}{\cancel{4}}}m_e^{\overset{6}{\cancel{8}}} \times 2.9544619 \times 10^{-165}}{h^4\ \cancel{G^2}\ \cancel{m_e^{\;2}}} \qquad (15)$$

Solving (13) for $G\,m_e^3$

$$Gm_e^3 = \frac{(860.79640)^2\, h^2\, R_\infty}{19.739209(36.144594 \times 10^{44})} \tag{16}$$

Substituting (16) in (15)

$$\lambda = \frac{(36.144594 \times 10^{44})^{\overset{2}{4}}(860.79640)^4 \cancel{h^4} R_\infty^2 (2.9544619 \times 10^{-165})}{(19.739209)^2 \cancel{(36.144594 \times 10^{44})^2} \cancel{h^4}}$$

$$\lambda = 5.4388537 \times 10^{-65}\, R_\infty^2 \text{ cm.}^{-2} \tag{17}$$

Thus it is seen that λ is a function of R_∞ alone.

It will now be remembered that U_L is to represent the number of centimeters per "natural centimeter".

$$U_L = \frac{1}{\sqrt{5.4388537 \times 10^{-65} \times R_\infty^2 \text{cm.}^{-2}}} \tag{18}$$

Now substituting the 1929 experimental value as given by Birge in (18)

$$U_L = \frac{1}{R_\infty \sqrt{54.388537 \times 10^{-66}}} = \frac{10^{33}}{1.0973742 \times 10^5 \sqrt{54.388537}}$$

$$U_L = 1.2356389 \times 10^{27} \text{ cm./natural centimeter} \tag{19}$$

The next unit of measure to be considered is that of time. Recall that our unit of time is defined as the length of time required for light to travel a distance equal to our unit length, that is the "radius" of the universe, and that it is called a natural second. This is equivalent to saying that the speed of light in vacuo is 1 nat.-cm./nat.-sec.

Let U_T be our unit of time. By definition

$$U_T = \frac{U_L}{c} \tag{20}$$

$$U_T = \frac{(1.2356389 \times 10^{27} \text{ cm./nat.-cm.})(1 \text{ nat.-cm/nat.-sec.})}{2.99796 \times 10^{10} \text{ cm./sec.}} \qquad (21)$$

The number 2.99796×10^{10} cm./sec. is the experimental value as given by Birge in 1929 for the speed of light, and is the natural constant which holds second place in the experimental precision of its determination.

$$U_T = 4.121599 \times 10^{16} \text{ sec./nat.-sec.}^{*} \qquad (22)$$

The next unit of measure to be determined is the unit of mass. On page 2 our unit of mass was defined as the mass of the universe, and was there called a natural gram. It is a little more laborious to determine the numerical value of the unit mass in grams. The methods or means for determining numerical values for U_L and U_T were limited because λ can not only be tied to the radius of the universe but can be expressed with even more precision than is necessary; c also is a constant that is known to a high degree of precision and may be utilized in determining U_T. However, U_M must be calculated from various constants of a low order of precision and a weighted mean of the results taken, to compensate for their individual poverty of precision. It is, therefore, proposed to find U_M from each of these constants and then find the weighted mean.

I. U_M will first be found using the experimental value of m_e, the mass of the electron. Birge's 1929 value for the mass of the electron is $(9.035_{10} \pm 0.010) \times 10^{-28}$ grams (spectroscopic method) and $(8.994_{25} \pm 0.014) \times 10^{-28}$ grams (deflection method). We shall use as the experimental value of the mass of the electron, the weighted average of these two values. This average is $(9.0215 \pm 0.0081) \times 10^{-28}$ grams. Substituting this value in (2) page 12.

*Determination of the proper number of significant figures is postponed until experimental errors have been discussed.

$$\frac{M_p}{9.0215 \times 10^{-28}} = 1834.1133 \qquad (23)$$

$$M_p = 1834.1133 \times 9.0215 \times 10^{-28} \qquad (24)$$

The mass of the universe will equal the sum of the masses of all the protons and the masses of all the electrons in the universe. According to Eddington's theory there are in our finite universe 136×2^{256} protons and the same number of electrons, or their equivalent in neutrons, photons, positrons, mesons, etc.

$$U_M = 136 \times 2^{256} \times M_p + 136 \times 2^{256}\, m_e = 136 \times 2^{256}(M_p + m_e)$$

$$= 136 \times 2^{256}(1834.1133 \times 9.0215 \times 10^{-28} + 9.0215 \times 10^{-28})$$

$$= 136 \times 2^{256} \times 9.0215 \times 10^{-28}(1834.1133 + 1)$$

$$= 136 \times 2^{256} \times 9.0215 \times 10^{-28}(1835.1133)$$

$$U_M = 2.6070804 \times 10^{55} \text{ gm., mat.-gm.} \qquad (25)$$

II. Next we shall determine U_M using the experimental value of $M_p = (1.6608 \pm 0.0017) \times 10^{-24}$. This value is given by Birge in his 1929 Table.

Substituting this value in (2) page 12

$$\frac{1.6608 \times 10^{-24}}{m_e} = 1834.1133 \qquad (26)$$

$$m_e = \frac{1.6608 \times 10^{24}}{1834.1133} = 9.0550567 \times 10^{-28} \text{ gm.}$$

Assume again that the mass of the universe is equal to the sum of the masses of 136×2^{256} electrons and an equal number of protons

Thus $\quad U_M = 136 \times 2^{256}(M_p + m_e)$

$U_M = 136 \times 2^{256}(1.6603 \times 10^{-24} + 9.0550567 \times 10^{-28})$

$$U_M = 2.6168089 \times 10^{55} \text{ gm./nat.-gm.} \tag{27}$$

III. We shall find another value for U_M when we use the experimental value of G. Birge's 1929 value is

$$6.664 \times 10^{-8} \text{ dyne cm.}^2/\text{gm.}^2$$

This value is the result of Heyl's determination of G. The units may be expressed as

$$6.664 \times 10^{-8} \cancel{\text{gm.}} \text{ cm.}^3/\text{gm.}^{\cancel{2}} \text{ sec.}^2$$

It is necessary at this point to consider carefully (4) page 12. The equation is

$$\frac{1}{\sqrt{\lambda}} = \frac{2G\left[136(2)^{256}(1835.1133)m_e\right]}{\pi c^2} \tag{28}$$

But by Eddington's work the reciprocal of the square root of λ is equal to the "radius" of the universe, and we have selected the "radius" of the universe to be our unit length! Therefore we may say that the left member of (28) equals 1 and write the equation as

$$1 = \frac{2G\left[136(2)^{256}(1835.1133)m_e\right]}{\pi c^2} \tag{29}$$

But the expression in brackets $\left[136(2)^{256}(1835.1133)m_e\right]$ is equal to the mass of the universe in grams, which we are representing by U_M, and c^2, the velocity of light, is unity squared. Therefore we may write (29) as

$$1 \text{ nat.-cm.} = \frac{2 \text{ G } U_M \text{ gm.}}{\pi(1)^2 \text{ nat.-cm.}^2/\text{nat.-sec}^2} \tag{30}$$

Solving for U_M, substituting in the experimental value of G and putting in the appropriate conversion factors, $U_T{}^2$ and $U_L{}^3$ we have

$$U_M \text{ gm.} = \frac{(\pi \frac{\text{nat-cm}^3}{\text{nat-sec}^2})(1.2356389 \times 10^{27} \frac{\text{cm}}{\text{nat-cm}})^3}{2(6.664 \times 10^{-8} \frac{\text{cm}^3}{\text{gm-sec}^2})(4.1215990 \times 10^{16} \frac{\text{sec}}{\text{nat-sec}})^2} \qquad (31)$$

Thus U_M which is the number of grams in a nat.-gm. turns out to be

$U_M = 2.6178135 \times 10^{55}$ gm./nat.-gm.

IV. Another value for U_M may be found by using the experimental value for e. Birge's 1929 value for e is $(4.770 \pm 0.005) \times 10^{-10}$ abs. e.s.units. It is convenient to express e in centimeters. A thing like this may be done since the units of "dielectric constant", k, are purely arbitrary so long as units of permeability, μ, are likewise so chosen that $1/\sqrt{k\mu}$ has the dimensions of a velocity. Consider the expression for Coulomb's law

$$F = \frac{qq'}{kr^2}$$

Let k = 1 sec.^{2}/gm.-cm. (in vacuo)

r is in centimeters and F is expressed in dynes (or gm. cm./sec^2).

We have then that

$$\cancel{\frac{\text{gm.cm.}}{\text{sec.}^2}} \times \cancel{\frac{\text{sec.}^2}{\text{gm.cm.}}} \times \text{cm.}^2 = qq'$$

$$qq' = \text{cm.}^2$$

$$\text{or} \qquad q = \text{cm.}$$

Therefore we may say that

$$e = (4.77 \pm 0.005) \times 10^{-10} \text{ cm.}$$

Equation (3) page 12 may be written as

$$e^2 = Gm_e^2\ k\ 4.1989714 \times 10^{42} \qquad (32)$$

From (30) page 17 G may be expressed in natural units as

$$G = \frac{\pi}{2}\,\frac{(\text{nat-cm})^3}{(\text{nat-gm})(\text{nat-sec})^2}$$

Therefore (32) is

$$e^2 = \frac{\pi}{2}\,m_e^{\,2}\,k\;4.1989714 \times 10^{42} \qquad (32A)$$

Converting e^2 into "natural" units and substituting in (32A)

$$\frac{(4.770 \times 10^{-10})^2}{(1.2356389 \times 10^{27})^2} = \frac{\pi}{2}\,k\,m_e^{\,2}\;4.1989714 \times 10^{42}$$

or $$2.2593880 \times 10^{-116} = k\,m_e^{\,2} \qquad (33)$$

where everything is now in natural units.

It is now necessary to find numerical values for $m_e^{\,2}$ and k.

Consider (2) page 12

$$M_p = 1834.1133\,m_e$$

Since there are 136×2^{256} protons and the same number of electrons then the mass of all of them must be

$$136 \times 2^{256}(1834.1133\,m_e + m_e) = 1 \text{ nat.-gm.}$$

$$\text{or } m_e = \frac{1}{(1835.1133)(136 \times 2^{256})}$$

$$m_e = 3.4603443 \times 10^{-83} \text{ nat.-gm.} \qquad (33A)$$

Substituting this value into (33) gives

$$k = 1.8869144 \times 10^{49} \qquad (34)$$

But in c.g.s. units

$$k = 1\,\frac{\text{sec.}^2}{\text{gm.-cm.}} \qquad (34A)$$

Converting this expression to "natural units" we obtain

$$k = \frac{\left(1\frac{\cancel{sec.^2}}{\cancel{gm-cm}}\right)(1.2356389 \times 10^{27})\frac{\cancel{cm.}}{nat-cm}\ U_M\ \frac{\cancel{gm.}}{nat-gm}}{(4.1215990 \times 10^{16})^2\ \frac{\cancel{sec.^2}}{nat-sec^2}}$$

or $k = 7.2737790 \times 10^{-7}\ U_M$. In the equation, k is in natural units and U_M is in gm./nat-gm.. Substituting this value into (34) and solving for U_M

$U_M = 2.5941319 \times 10^{55}$ gm./nat.-gm.

V. It is also possible to find a value for U_M from the experimental value for Planck's constant. Birge's 1929 value for Planck's constant is $h = (6.547 \pm 0.008) \times 10^{-27}$ erg-sec. or $h= 6.547 \times 10^{-27}\ \frac{gm-cm^2}{sec.}$.

We may convert these units to natural units

$$h = \frac{\left(6.547 \times 10^{-27}\ \frac{\cancel{gm-cm^2}}{\cancel{sec}}\right)\left(4.1215990 \times 10^{16}\ \frac{\cancel{sec}}{nat-sec.}\right)}{\left(U_M\ \frac{\cancel{gm.}}{nat-gm.}\right)\left(1.2356389 \times 10^{27}\ \frac{\cancel{cm.}}{nat-cm.}\right)}$$

$$h = \frac{1.7673597 \times 10^{-64}}{U_M} \qquad (35)$$

$$h = \frac{137(2\pi e^2)}{ck}$$ also in natural units (page 7, eq. (1))

Therefore $$\frac{137(2\pi e^2)}{c\ k} = \frac{1.7673597 \times 10^{-64}}{U_M} \qquad (36)$$

From (32A) page 19 $e^2 = \frac{\pi}{2} m_e^2\ k\ 4.1989714 \times 10^{42}$

Substituting this value into (36)

$$\frac{137(2\pi)\frac{\pi}{2} m_e^2\ \cancel{k}\ 4.1989714 \times 10^{42}}{c\ \cancel{k}} \quad \frac{1.7673597 \times 10^{-64}}{U_M} \qquad (37)$$

And from (33A) page 19 $m_e = 3.4603443 \times 10^{-63}$.

Substituting the value, 1 nat-cm./nat-sec., for c into (37)

$$137(2\pi)\frac{\pi}{2}(3.4603443 \times 10^{-83})^2\ 4.1989714 \times 10^{42} = \frac{1.7673597 \times 10^{-64}}{U_M} \qquad (38)$$

Solving: $U_M = 2.5996989 \times 10^{55}$ gm./nat.-gm.

We have found five different values for U_M. It is now necessary to discover what error is involved in each value. When U_M was calculated from the mass of the electron, an experimental value of $(9.0215 \pm 0.0081) \times 10^{-28}$ was used. The percentage error is

$$E\% = \frac{0.0081 \times 100}{9.0215} = 0.090\%$$

Therefore 0.090% of the value of U_M is the actual error.

$$P.E. = 0.00090 \times 2.6070804 = 0.0023.$$

Therefore $U_M = (2.6071 \times 0.0023) \times 10^{55}$

The error involved in finding U_M from the mass of the proton

$$E\% = \frac{0.0017 \times 100}{1.6608} = 0.10\%$$

0.10% of 2.6168080 = 0.0026

Therefore $U_M = (2.6168 \pm 0.0026) \times 10^{55}$

U_M was found from the experimental value of $(6.664 \pm 0.002) \times 10^{-8}$

The percentage error is

$$E\% = \frac{0.002 \times 100}{6.664} = 0.03\%$$

However to find the actual error it is necessary to allow for errors in U_L and U_T, since these two quantities were used in this determination of U_M.

The error involved in U_L is the same as the error in the Rydberg constant. The value of the Rydberg constant used was

$$R_\infty = (1.0973742 \pm 0.0000006) \times 10^5$$

The percentage error here is

$$E\% = \frac{0.0000006 \times 100}{1.0973742} = 0.000055\%$$

The value of c used in determining U_T was $(2.99796 \pm 0.00004) \times 10^{10}$

The percentage error is

$$E\% = \frac{0.00004 \times 100}{2.99796} = 0.0013\%.$$

This must be doubled since U_T enters as a factor twice in (31).

The error in U_L is 68×10^{19} and is therefore completely negligible. So we may consider that the error involved in finding U_M in this case arises out of the errors of G and of U_T.

The square root of the sum of the squares of the errors from G and from U_T will be the percentage error of U_M

$$E\% = \sqrt{(0.0026)^2 + (0.03)^2}$$

$$E\% = 0.030\%$$

That is, the error in U_T is also negligible

$$0.030\% \text{ of } 2.6178135 = 0.00078$$

$$U_M = 2.61781 \pm 0.00078$$

The experimental value of e was $(4.770 \pm 0.005) \times 10^{-10}$

$$E\% = \frac{0.005 \times 100}{4.770} = 0.10\%$$

In the computation the experimental value was squared therefore the percentage error was doubled. The error due to U_T is negligible.

$$2 \times 0.10\% = 0.20\%$$

Error in U_M is therefore 0.20% of 2.5941319 = 0.0052

$$U_M = (2.5941 \pm 0.0052) \times 10^{55}$$

The percentage error in the experimental value of h, (6.547±0.008), is

$$E\% = \frac{0.008 \times 100}{6.547} = 0.12\%$$

The error involved in U_T is still negligible. Thus the percentage error in U_M is

$$E\% = 0.12\%$$

$$0.12\% \text{ of } U_M = 0.0032$$

Thus $U_M = (2.5997 \pm 0.0032) \times 10^{55}$

For our convenience the five values of U_M are tabulated here in grams per nat.-gram.

TABLE III.

From m_e	$(2.6071 \pm 0.0023) \times 10^{55}$
From M_p	$(2.6168 \pm 0.0026) \times 10^{55}$
From G	$(2.61781 \pm 0.00078) \times 10^{55}$
From e	$(2.5941 \pm 0.0052) \times 10^{55}$
From h	$(2.5997 \pm 0.0032) \times 10^{55}$

Our conversion factor U_M will be the weighted mean of the five different values determined.

The comparative weights of the different values are found by comparing the ratios of the reciprocal of the squares of the different errors.

Thus

$$\frac{1}{(.0023)^2} : \frac{1}{(.0026)^2} : \frac{1}{(.00078)^2} : \frac{1}{(.0052)^2} : \frac{1}{(.0032)^2}$$

or

$$\frac{1}{0.0000053} : \frac{1}{0.0000068} : \frac{1}{0.00000071} : \frac{1}{0.000029} : \frac{1}{0.000010}$$

These ratios may be expressed as

$$6 : 4 : 41 : 1 : 3$$

In other words the value having the least error would have the greatest

weight. The weighted mean of U_M then is

$$U_M = \frac{6(2.6071) + 4(2.6168) + 41(2.61781) + 1(2.5941) + 3(2.5997)}{6 + 4 + 41 + 1 + 3}$$

$U_M = 2.61515 \pm 0.00069 \times 10^{55}$ gm./nat.-gm.

The error 0.0069 is found from

$$U_M = \sqrt{\left[\frac{6}{55}(0.0025)\right]^2 + \left[\frac{4}{55}(0.0026)\right]^2 + \left[\frac{41}{55}(0.00078)\right]^2 + \left[\frac{1}{55}(0.0052)\right]^2 + \left[\frac{3}{55}(.0032)\right]^2}$$

$U_M = 0.00069$

Our "natural units" of length, time, and mass are then:

$$U_L = (1.23563890 \pm 0.00000068) \times 10^{27} \text{ cm./nat.-cm.} \quad (40)$$

$$U_T = (4.121599 \pm 0.000054) \times 10^{16} \text{ sec./nat.-sec.} \quad (40)$$

$$U_M = (2.61515 \pm 0.00069) \times 10^{55} \text{ gm./ nat.-gm.} \quad (40)$$

The percentage errors are:

$$E\% \text{ for } U_L = \frac{0.00000068 \times 100}{1.23563890} = 0.000055\% \quad (41)$$

$$E\% \text{ for } U_T = \frac{0.000054 \times 100}{4.121599} = 0.0013\% \quad (41)$$

$$E\% \text{ for } U_M = \frac{0.00069 \times 100}{2.61515} = 0.0026\% \quad (41)$$

Now it remains for us to apply these fundamental conversion factors (40) to the values of the constants as expressed in "natural units". It will be seen that the "natural units" cancel out and our results are expressed in units of the c.g.s. system.

By definition, c is $1 \frac{\text{nat.-cm.}}{\text{nat.-sec.}}$

$$c = 1\frac{\cancel{\text{nat-cm}}}{\cancel{\text{nat-sec}}}\;\frac{1.2356389 \frac{\text{cm.}}{\cancel{\text{nat-cm}}} \times 10^{27}}{4.12199 \frac{\text{sec}}{\cancel{\text{nat-sec}}} \times 10^{16}}$$

Therefore $c = 2.99796 \times 10^{10}$ cm./sec.

The error would be found by finding the square root of the sum of the squares of the percentage errors involved in the units except that there is a negligible error in U_L. Therefore

$$E\% = 0.0013\%$$

Therefore the error in c is

$$0.0013\% \text{ of } 2.99796 = 0.000039$$

$$c = (2.997960 \pm 0.000039) \times 10^{10} \text{ cm./ sec.} \qquad (42)$$

$$\lambda = (1 \text{ nat.-cm.})^{-2}(1.2356389 \times 10^{27} \text{ cm./nat.-cm.})^{-2}$$

$$\lambda = 6.5496313 \times 10^{-55} \text{ cm.}$$

The error is due to the error in U_L. That is

$$E\% = 0.00011\%$$

$$0.00011\% \text{ of } 6.5496313 = 0.0000072$$

$$\lambda = (6.5496313 \pm 0.0000072) \times 10^{-55} \text{ cm.} \qquad (43)$$

$$m_e = 3.4603433 \times 10^{-83} \text{ nat.-gm.} \times 2.61515 \times 10^{55} \text{ gm./nat.-gm.}$$

$$m_e = 9.0493 \times 10^{-28} \text{ gm.}$$

The error is due to that in U_M. That is 0.026%

$$0.026\% \text{ of } 9.0493 = 0.0024$$

$$m_e = (9.0493 \pm 0.0024) \times 10^{-28} \text{ gm.} \qquad (44)$$

M_p = .834.1133 m_e from (2) page 12

But $m_e = (9.0493 \pm 0.0024) \times 10^{-28}$ gm. from (44)

Therefore $M_p = 1834.1133 \times 9.0493 \times 10^{-28}$

$$M_p = 1.65974 \times 10^{-24} \text{ gm.}$$

The percentage error is the same as that in m_e, or 0.026%

$$0.026\% \text{ of } 1.65974 = 0.00043$$

$$M_p = (1.65974 \pm 0.00043) \times 10^{-24} \text{ gm.} \qquad (45)$$

Substituting $k = \frac{1.2356389 \times 10^{27} \times (2.61515 \times 10^{55})}{4.121599 \times 10^{16\ 2}} = 1.90220 \times 10^{49}$

and $m_e = 3.4603443 \times 10^{-83}$ in (32A) page 19.

$e = 3.8746854 \times 10^{-37}$ nat.-cm. where $E\% = 0.031$ (45A)

$e = 3.8746854 \times 10^{-37}$ nat.-cm $\times 1.2356389 \times 10^{27}$ cm./nat.-cm.

$e = 4.7877120 \times 10^{-10}$ cm.

The percentage error is 0.000083% due to U_L and 0.0013% due to U_T, both of which are negligible. The total error is therefore 0.013% due to U_M, or 0.00063×10^{-10}.

$e = (4.78771 \pm 0.00063) \times 10^{-10}$ c.g.s.e.s.units (46)

$$G = \frac{\pi}{2} \frac{\text{nat-cm}^3}{\text{nat-gm}(\text{nat-sec})^2} \quad \text{from (30) page 17}$$

$$G = \frac{\pi}{2} \frac{\text{nat-cm}^3 (1.2356389 \times 10^{27})^3 \text{cm}^3}{\text{nat-gm}(\text{nat-sec})^2 (2.61515 \times 10^{55}) \frac{\text{gm}}{\text{nat-gm}} (\text{nat-cm})^3 (4.121599 \times 10^{16})^2 \frac{\text{sec}^2}{\text{nat-sec}^2}}$$

$$G = 6.6752335 \times 10^{-8} \frac{\text{cm}^3}{\text{gm-sec}^2}$$

The error as before must be calculated by finding the square root of the sum of the squares of the errors involved, two of which are, however, negligible.

$E\% = 0.026\%$

0.026% of 6.6752335 = 0.0017

$$G = (6.6752 \pm 0.0017) \times 10^{-8} \frac{\text{cm}^3}{\text{gm-sec}^2} \quad (47)$$

$$h = \frac{e^2(860.79640)}{c\ k} \quad \text{from (1) page 12 .}$$

But $e = 4.78771 \times 10^{-10}$ cm. and $c = 2.99796 \times 10^{10}$ cm/sec. and $k = 1 \frac{\text{sec}^2}{\text{gm-cm}}$

Therefore $h = \dfrac{(4.78771 \times 10^{-10})^2 (860.79640)\ cm^2\ sec\ gm\ \cancel{cm}}{2.99796 \times 10^{10} \times 1\ \cancel{cm}\ sec^2}$

$$h = 6.5815872 \times 10^{-27}\ gm\ cm^2/sec.$$

This error is due to the error in e; the error in c is negligible in comparison

$$E\% = 0.026\%$$

$$0.026\% \text{ of } 6.5815872 = 0.0017$$

$$h = (6.5816 \pm 0.0017) \times 10^{-27}\ gm.\ cm^2/sec.$$

The physical constants as we have calculated them are

$$c = (2.997960 \pm 0.000039) \times 10^{10}\ cm/sec$$

$$m_e = (9.0493 \pm 0.0024) \times 10^{-28}\ gm.$$

$$M_p = (1.65974 \pm 0.00043) \times 10^{-24}\ gm.$$

$$G = (6.6752 \pm 0.0017) \times 10^{-8}\ cm^3/gm\text{-}sec^2$$

$$e = (4.78771 \pm 0.00063) \times 10^{-10}\ cm.$$

$$h = (6.5816 \pm 0.0017) \times 10^{-27}\ gm\text{-}cm^2/sec$$

$$= (6.5496313 \pm 0.0000072) \times 10^{-55}\ cm^{-2}$$

How do these values compare with Birge's values?

TABLE IV.

	Birge (1929)	Calculated
c	$(2.99796 \pm 0.00004) \times 10^{10}$	$(2.997960 \pm 0.000040) \times 10^{10}$
m_e	$(9.0215 \pm 0.0081) \times 10^{-28}$	$(9.0493 \pm 0.0024) \times 10^{-28}$
M_p	$(1.66089 \pm 0.0017) \times 10^{-24}$	$(1.65974 \pm 0.00043) \times 10^{-24}$
G	$(6.664 \pm 0.002) \times 10^{-8}$	$(6.6752 \pm 0.0017) \times 10^{-8}$
e	$(4.770 \pm 0.005) \times 10^{-10}$	$(4.78771 \pm 0.00063) \times 10^{-10}$
h	$(6.547 \pm 0.008) \times 10^{-27}$	$(6.5816 \pm 0.0017) \times 10^{-27}$
λ		$(6.5496313 \pm 0.0000072) \times 10^{-55}$

TABLE V.

	Birge(1929)	Calculated from 1929 experimental values	Birge (1941)
c	2.99796×10^{10}	$2.99796 \pm 0.00004 \times 10^{10}$	$2.99776 \pm 0.00004 \times 10^{10}$
m_e	9.0215×10^{-28}	$9.0493 \pm 0.0024 \times 10^{-28}$	$9.10660 \pm 0.0032 \times 10^{-28}$
M_p	1.66089×10^{-24}	$1.65974 \pm 0.00043 \times 10^{-24}$	$1.672482 \pm 0.00031 \times 10^{-24}$
G	6.664×10^{-8}	$6.6752 \pm 0.0017 \times 10^{-8}$	$6.670 \pm 0.005 \times 10^{-8}$
e	4.770×10^{-10}	$4.78771 \pm 0.00063 \times 10^{-10}$	$4.80251 \pm 0.0010 \times 10^{-10}$
h	6.547×10^{-27}	$6.5816 \pm 0.0017 \times 10^{-27}$	$6.6242 \pm 0.0024 \times 10^{-27}$
λ		$6.5496313 \pm 0.0000072 \times 10^{-55}$	

It is seen from the table that the calculated value of c agrees with 1929 experimental but is greater than 1941 experimental value. The calculated value for m_e is greater than the 1929 experimental value but less than the 1941 experimental. It would seem reasonable then to say that if the 1941 experimental value is a better value for m_e than the 1929 value then the calculated value is a better value than 1929 value. A study of all the constants in the table on page 28 results in the same conclusions as were reached for m_e, except for M_p. That is, all calculated values are between the 1941 experimental value and the 1929 experimental values except c, G, and M_p. There is no change for c from the 1929 value because c was utilized in our standard of time. The calculated value of G is not only greater than the 1929 value; it is slightly greater even than the 1941 value. M_p alone seems to veer in the wrong direction.

Since the majority of the calculated values show improvement over the 1929 experimental values, it would seem that we have evidence that the system of calculating these values is theoretically sound. Now since we have improved the 1929 values and in that way, justified the procedure

adopted in this thesis, the logical procedure is to repeat the calculations, using the 1941 experimental values. We shall then have the best possible values for the seven physical constants considered in this thesis consistent with present data and it will be the work of the future to pass judgment upon them. The experimental values to be used now are from Birge's tables for 1941. They are

$c = (2.99776 \pm 0.00004) \times 10^{10}$ cm/sec. (49)

$m_e = (9.10660 \pm 0.0032) \times 10^{-28}$ gm. "

$M_p = (1.672482 \pm 0.00031) \times 10^{-24}$ gm. "

$G = (6.670 \pm 0.005) \times 10^{-8}$ cm^3/gm-sec^2 "

$e = (4.80251 \pm 0.0010) \times 10^{-10}$ cm. "

$h = (6.6242 \pm 0.0024) \times 10^{-27}$ gm-cm^2/sec. "

$R_\infty = (1.09737303 \pm 0.0000005) \times 10^{5}$ cm^{-1} "

As before, a unit length is defined as equal to the Einstein hyper-radius and is determined by finding the reciprocal of the square root of λ.

$\lambda = 5.4388537 \times 10^{-65}\ R_\infty^2$ from (17)

Now $R_\infty = (1.09737303 \pm 0.0000005) \times 10^{5}$

$\lambda = 5.4388537 \times 10^{-65} \times (1.09737303 \times 10^5)^2$

$\lambda = 6.5496172 \times 10^{-55}$ cm^{-2}

$$\frac{1}{\sqrt{\lambda}} = \frac{1}{\sqrt{6.5496172 \times 10^{-55}}} = U_L = 1.2356403 \times 10^{27} \text{ cm.}$$

The percentage error is due to that of R_∞.

$$E\% = \frac{.0000005}{1.09737303} \times 100 = 0.000045\%$$

0.000045% of $1.2356403 = 0.00000056$

$U_L = (1.2356403 \pm 0.00000056) \times 10^{27}$ cm./mat-cm. (50)

Our unit time U_T is the time necessary for light to travel 1 natural centimeter

$$= 1 \frac{\text{nat-cm.}}{\text{nat-sec.}}$$

$$1 \text{ nat-sec.} = \frac{1 \text{ nat-cm.}}{c} \quad \text{where } c = 2.99776 \times 10^{10} \text{ cm./sec.}$$

from (49) page 29

$$1 \text{ nat-sec.} = \frac{1.2356403 \times 10^{27}}{2.99776 \times 10^{10}}$$

$$1 \text{ nat-sec.} = 4.1218786 \times 10^{16} \text{ cm./nat-sec.}$$

The error is due to that in U_L and c. The error in U_L = 0.000045% and

$$E\% \text{ in c} = \frac{0.00004 \times 100}{2.99776} = 0.0013\%$$

The square root of the sum of the squares of these two values is the percentage error in U_T

$$E\% = \sqrt{(0.000045)^2 + (0.00133)^2}$$

$$E\% = \sqrt{(0.0000000020 + 0.00000178}$$

$$E\% = \sqrt{0.000001782}$$

$$E\% = 0.0013\%$$

0.0013% of U_T is

0.0013% of 4.1218786 = 0.000055

$$U_T = (4.121879 \pm 0.000055) \times 10^{16} \text{ sec./nat-sec.} \qquad (51)$$

As before, a value for unit mass will be determined from the experimental values of several constants and the weighted mean of these results will be the value of U_M, our unit mass. Unit mass is defined as being equal to the mass of the universe.

Our first value of U_M is found using the experimental value of m_e.

From (49) $m_e = (9.10660 \pm 0.0032) \times 10^{-28}$.

This value for m_e is substituted into (2) page 12.

$$\frac{M_p}{9.10660 \times 10^{-28}} = 1834.1133 \qquad (52)$$

$$M_p = 1834.1133(9.10660 \times 10^{-28})$$

The mass of the universe equals the sum of the masses of all the electrons and protons in the universe. According to Eddington there are 136×2^{256} electrons and the same number of protons in the universe.

Therefore $$U_M = 136 \times 2^{256}(M_p + m_e) \qquad (52A)$$

$$U_M = 136 \times 2^{256}\left[1834.1133(9.10660 \times 10^{-28}) + 9.10660 \times 10^{-28}\right]$$

$$U_M = 2.6317034 \times 10^{55} \text{ gm/nat-gm.}$$

The error is due to the error in the experimental value of m_e. This is

$$E\% = \frac{0.0032 \times 100}{9.10660} = 0.035\%$$

0.035% of U_M is

0.035% of 2.6317034 = 0.00092

$$U_M = (2.63170 \pm 0.00092) \times 10^{55} \text{ gm/nat.-gm.} \qquad (53)$$

Using the same procedure we will find a value for U_M using the experimental value of M_p in (49). That value is

$$M_p = (1.672482 \pm 0.00031) \times 10^{-24}$$

Substituting this value into (2) page 12.

$$\frac{1.672482 \times 10^{-24}}{m_e} = 1834.1133$$

$$m_e = \frac{1.672482 \times 10^{-24}}{1834.1133}$$

Substituting this value for m_e and the experimental value of M_p into (52A)

$$U_M = 136 \times 2^{256}\left(\frac{1.672482 \times 10^{-24}}{1834.1133} + 1.672482 \times 10^{-24}\right)$$

$U_M = 2.6352145 \times 10^{55}$ gm/ nat.-gm.

The error in U_M is due to that in the experimental value of M_p.

This is

$$E\% = \frac{0.00031 \times 100}{1.672482} = 0.019\%$$

0.019% of U_M = 0.019% of 2.6352145 = 0.00049

$$U_M = (2.63521 \pm 0.00049) \times 10^{55} \text{ gm./ nat-gm.} \qquad (54)$$

A value for U_M is now found from the experimental value of G.

From (49) $G = (6.670 \pm 0.005) \times 10^{-8}$ cm^3/gm-sec^2

Substituting this value and the values of U_L and U_T into (30) we have

$$1 \text{ nat-cm.} = \frac{2(6.670\times10^{-8})\frac{cm^3}{gm-sec^2}(U_M\frac{gm}{nat-gm})4.1218786\times10^{16})^2\frac{sec^2}{nat-set^2}}{(1.2356403\times10^{27})^3 \text{ cm}^3/\text{nat-cm}^3}$$

Solving for U_M

$$U_M = 2.6150493 \times 10^{55}$$

The error in U_M is due to the error in U_T, U_L and in the experimental value of G.

The error in G is $\frac{0.005 \times 100}{6.670} = 0.075\%$

The error in U_L = 0.000045%

The error in U_T = 0.0013%

Percentage error in $U_M = \sqrt{(0.000045)3^{\;2} + (0.0013)2^{\;2} + (0.075)^2}$

E% = 0.075%

0.075% of 2.6150493 = 0.0022

$U_M = (2.6150 \pm 0.0022) \times 10^{55}$ gm./nat.-gm (55)

Now the value of U_M using the experimental value of

$e = (4.80251 \pm 0.0010) \times 10^{-16}$ cm. will be obtained.

Converting e to "natural units" and substituting in (32) page 18 we get

$$\left(\frac{4.80251 \times 10^{-10} \text{ cm.}}{1.2356403 \times 10^{27} \frac{\text{cm}}{\text{nat-cm}}}\right)^2 = \frac{\pi}{2} m_e^2 \, k \, 4.1989714 \times 10^{42} \qquad (56)$$

But $m_e = 3.4663443 \times 10^{-83}$ nat-gm. from (33A)

Therefore (56) becomes

$$\left(\frac{4.80251 \times 10^{-10}}{1.2356403 \times 10^{27}} \text{ nat-cm.}\right)^2 = \frac{\pi}{2}(3.4663443 \times 10^{-83})^2 k 4.1989714 \times 10^{42} \qquad (57)$$

But from (34A) $\qquad k = 1 \frac{\text{sec}^2}{\text{gm-cm}}$

or in natural units

$$k = \frac{1 \frac{\cancel{\text{sec}^2}}{\cancel{\text{gm-cm}}} (1.2356403 \times 10^{27}) \frac{\cancel{\text{cm}}}{\text{nat-cm}} \; U_M \frac{\cancel{\text{gm}}}{\text{nat-gm.}}}{(4.1218786 \times 10^{16}) \frac{\cancel{\text{sec}^2}}{\text{nat-sec}^2}}$$

Substituting this value for k into (57) and solving for U_M

$$U_M = 2.6299611 \times 10^{55}$$

The error in e is $\dfrac{0.001 \times 100}{4.80251} = 0.021\%$

The error in $U_L = 0.00005\%$

The error in $U_T = 0.0013\%$

Since e was squared the error due to e was doubled. Since U_L was cubed the error due to U_L was tripled. The error in U_M is equal to the square root of the sum of the squares of the individual errors

$$E\% = \sqrt{(0.0416)^2 + (0.00015)^2 + (0.0013)^2}$$

$$E\% = \sqrt{0.00173 + 0.0000000225 + 0.00000177}$$

$$E\% = 0.042\%$$

0.042% of 2.6299611 = 0.0011

$$U_M = (2.6400 \pm 0.0011) \times 10^{55} \text{ gm/nat-gm.} \qquad (58)$$

Finally a value for U_M is found from the experimental value of h

$h = (6.6242 \pm 0.0024) \times 10^{-27}$ where the percentage error is

$$\frac{0.0024 \times 100}{6.6242} = 0.036\%$$

Converting h to "natural" units we obtain

$$h = \frac{(6.6242 \times 10^{-27} \cancel{\frac{\text{gm-cm}^2}{\text{sec}}})(4.1218786 \times 10^{16} \frac{\cancel{\text{sec}}}{\text{nat-sec}})}{(U_M \frac{\cancel{\text{gm}}}{\text{nat-gm}})(1.2356403 \times 10^{27} \frac{\cancel{\text{cm}}}{\text{nat-cm}})^2}$$

$$h = \frac{1.7883169 \times 10^{-64}}{U_M} \frac{\text{nat-cm}^2}{\text{nat-gm}} \frac{\text{nat-gm}}{\text{nat-sec}} \qquad (59)$$

Substituting (59) into (1) page 12

$$\frac{1.7883169 \times 10^{-64}}{U_M} \quad \frac{137(2 \quad e^2)}{c \ k}$$

From (32) $\qquad e^2 = \frac{\pi}{2} m_e{}^2 k \ 4.1989714 \times 10^{42}$

$$e^2 = \frac{\pi}{2}(3.4603443 \times 10^{-83})^2 \ 4.1989714 \times 10^{42} \ k$$

$$\frac{1.7883169 \times 10^{-64}}{U_M} = \frac{\cancel{k} \ 137(2\pi) \ \frac{\pi}{2}(3.4603443 \times 10^{-83})^2(4.1989714 \times 10^{42})}{1 \ \cancel{k}}$$

$$U_M = 2.6305260 \times 10^{55}$$

The error in U_M is due to the errors in the xperimental value of h, U_L , and U_T

$$E\% = \sqrt{(0.036)^2 + (0.00015)^2 + (0.0013)^2}$$

$$E\% = 0.036\%$$

0.036% of 2.6305260 = 0.00095

$$U_M = (2.63053 \pm 0.00095) \times 10^{55} \text{ gm/nat-gm.} \qquad (60)$$

We have now five different values of U_M, as shown in Table 6, expressed in gm/nat-gm.

TABLE 6 (61)

From m_e	$(2.63170 \pm 0.00092) \times 10^{55}$
M_p	$(2.63521 \pm 0.00049) \times 10^{55}$
G	$(2.6150 \pm 0.0022) \times 10^{55}$
e	$(2.6300 \pm 0.0011) \times 10^{55}$
h	$(2.63053 \pm 0.00095) \times 10^{55}$

Comparative weights are

$$\frac{1}{(0.00092)^2} : \frac{1}{(0.00049)^2} : \frac{1}{(0.0020)^2} : \frac{1}{(0.0011)^2} : \frac{1}{(0.00095)^2}$$

The weights are 12, 49, 3, 8, 11. Sum of the weights is 83

$$U_M = \frac{12}{83}(2.63170 \pm 0.00092) + \frac{49}{83}(2.63521 \pm 0.00049) + \frac{3}{83}(2.6150 \pm 0.0020)$$
$$+ \frac{8}{83}(2.6300 \pm 0.0011) + \frac{11}{83}(2.63053 \pm 0.00095)$$

$$U_M = 2.6328503 \times 10^{55}$$

$$\text{Error} = \delta U_M = \left\{\left[\frac{12}{83}(0.00092)\right]^2 + \left[\frac{49}{83}(0.00049)\right]^2 + \left[\frac{3}{83}(0.0020)\right]^2 + \left[\frac{8}{83}(0.0011)\right]^2 + \left[\frac{11}{83}(0.00095)\right]^2\right\}^{\frac{1}{2}}$$

$$\delta U_M = 0.00035$$

Therefore $U_M = (2.63285 \pm 0.00035) \times 10^{55}$ gm/nat-gm (62)

Our units for length, time and mass are

$U_L = (1.2356403 \pm 0.00000062) \times 10^{27}$ cm/nat-cm

$U_T = (4.121879 \pm 0.000055) \times 10^{16}$ sec/nat-sec

$U_M = (2.63285 \pm 0.00035) \times 10^{55}$ gm/nat-gm. (63)

and if we substitute these conversion factors into the expressions of the physical constants considered by this thesis we shall obtain in terms of the c.g.s. system the most probable values of these physical constants.

$$c = \frac{1 \frac{\text{nat-cm}}{\text{nat-sec}}(1.2356403 \times 10^{27}) \frac{\text{cm}}{\text{nat-cm}}}{4.1218786 \times 10^{16} \frac{\text{sec}}{\text{nat-sec}}} = 2.99776 \times 10^{10} \text{ cm/sec.}$$

The percentage error is $\sqrt{(0.00005)^2 + (0.0013)^2}$

$E\% = 0.0013\%$

0.0013% of $2.99776 = 0.00004$

$c = (2.99776 \pm 0.00004) \times 10^{10}$ cm/sec (64)

$\lambda = (1 \text{ nat-cm}^{-2})(1.2356403 \times 10^{27} \text{ cm/nat-cm})^2$

$\lambda = 6.5496163 \times 10^{-55} \text{ cm}^{-2}$

Percentage error equals $2 \times 0.00005\% = 0.00010\%$

0.00010% of $6.5496163 = 0.0000060$

$\lambda = (6.5496163 \pm 0.0000060) \times 10^{-55} \text{ cm}^{-2}$ (65)

$m_e = (3.4603433 \times 10^{-83} \text{ nat-gm})(2.6328503 \times 10^{55} \text{ gm/nat-gm.})$

$m_e = 9.1105649 \times 10^{-28}$ gm.

Percentage error equals 0.013%

0.013% of $9.1105649 = 0.0012$

$m_e = (9.1106 \pm 0.0012) \times 10^{-28}$ gm. (66)

$M_p = 1834.1133\ m_e = 1834.1133(9.1105649 \times 10^{-28})$ gm. $= 1.6709808 \times 10^{-24}$

The error is due to the error in m_e.

Percentage error equals 0.013%

0.013% of $1.6709808 = 0.00023$

$M_p = (1.67098 \pm 0.00023) \times 10^{-24}$ gm. (67)

From (32) $e = \sqrt{\frac{\pi}{2} m_e^2 \, k \, 4.1989714 \times 10^{42}}$ (68)

$$m_e^2 = (3.5603433 \times 10^{-83})^2$$

$$\text{and } k = \frac{1 \frac{\text{sec}^2}{\text{gm-cm}} (1.2356403 \times 10^{27}) \frac{\text{cm}}{\text{nat-cm}} (2.6328503 \times 10^{55}) \frac{\text{cm}}{\text{nat-cm}}}{(4.1218786 \times 10^{16}) \frac{\text{sec}}{\text{nat-sec}})^2}$$

$$k = 1.9148193 \times 10^{49} \frac{\text{nat-sec}^2}{\text{nat-cm nat-gm}}$$

Substituting these values of k and m_e^2 into (68) we obtain

$$e = \sqrt{\frac{\pi}{2}(3.4603433 \times 10^{-83})^2 (1.9148193 \times 10^{49})(4.1989714 \times 10^{42}}$$

$e = 3.8887911 \times 10^{-37}$ nat-cm. (69)

$e = 3.8887911 \times 10^{-37}$ nat-cm.$(1.2356403 \times 10^{27}$ cm/nat-cm.)

$e = 4.8051470 \times 10^{-10}$ cm. (69A)

The percentage error is the same as that in k except that the square root will half the error in e.

The error for $k = \sqrt{(0.013)^2 + (0.00005)^2 + (0.0013)^2}$

$$E\% = 0.013\%$$

The percentage error for $e = \frac{1}{2}(0.0133\%) = 0.0066\%$

0.0066% of 4.8051470 = 0.00031

$e = (4.80515 \pm 0.00031) \times 10^{-16}$ cm. (70)

$$G = \frac{\left(\frac{\pi}{2} \frac{\text{nat-cm}^3}{\text{nat-sec}^2 \text{ nat-gm}}\right)(1.2356403 \times 10^{27}) \frac{\text{cm}^3}{\text{nat-cm}^3}}{(4.1218786 \times 10^{16} \frac{\text{sec}}{\text{nat-sec}})^2 (2.6328503 \times 10^{55} \frac{\text{gm}}{\text{nat-gm}})}$$

$G = 6.6249042 \times 10^{-8}$

Percentage error is found by calculating the square root of the square of the errors of U_L, U_M, and U_T.

$$E\% = \sqrt{(0.00015)^2 + (0.0026)^2 + (0.00135)^2}$$

$$= \sqrt{0.0000000225 + 0.00000676 + 0.00000182} = \sqrt{0.0000086025}$$

$$E\% = 0.0029\%$$

$$0.0029\% \text{ of } 6.6249042 = 0.00019$$

$$G = (6.62490 \pm 0.00019) \times 10^{-8} \text{ cm}^3/\text{gm-sec}^2$$

$$h = \frac{e^2(860.79640)}{c\ k} \tag{71}$$

But $e = (4.8051470) \times 10^{-10}$ and $c = 2.99776 \times 10^{10}$ from (70) and (64)

Substituting these values into (71)

$$h = \frac{(4.8051470 \times 10^{-10})^2(860.79640)\ \text{cm}^2}{2.99776 \times 10^{10}\ \text{cm/sec}\ (1)\ \text{sec}^2/\text{gm-cm}}$$

$$h = (6.6300521) \times 10^{-27}\ \text{gm-cm}^2/\text{sec}$$

The error is due to the error in e and c

$$E\% = \sqrt{(0.0133)^2 + (0.0013)^2} = \sqrt{0.000176 + 0.00000169}$$

$$E\% = \sqrt{0.00017769}$$

$$E\% = 0.013\%$$

$$0.013\% \text{ of } 6.6300521 = 0.00088$$

$$h = (6.63005 \pm 0.00088) \times 10^{-27}\ \text{gm-cm}^2/\text{sec} \tag{72}$$

We may now add the results from the 1941 experimental values to the table on page 28. The last two columns will then appear as in table 7.

TABLE VII.

	Birge 1941 values	Computed from 1941 values
c	$(2.99776\pm0.00004)\times10^{10}$	$(2.9976\pm0.00004)\times10^{10}$
m_e	$(9.10660\pm0.0032)\times10^{-28}$	$(9.1106\pm0.0012)\times10^{-28}$
M_p	$(1.672482\pm0.00031)\times10^{-24}$	$(1.67098\pm0.00023)\times10^{-24}$
G	$(6.670\pm0.005)\times10^{-8}$	$(6.62490\pm0.00019)\times10^{-8}$
e	$(4.80251\pm0.0010)\times10^{-10}$	$(4.80515\pm0.00031)\times10^{-10}$
h	$(6.6242\pm0.0024)\times10^{-27}$	$(6.63005\pm0.00088)\times10^{-27}$
λ		$(6.5496163\pm0.000006)\times10^{-55}$

If the two columns of Table VII are compared the results of our calculations are seen to be that first the velocity of light is not changed. The reason for this is that it was used in the definition of our unit of time. Since c has been determined experimentally to extremely high precision, it is very fitting that it should be preserved in this way. The rest mass of the electron however is greater than experiment has so far been able to determine. The rest-mass of the proton, on the other hand, is slightly smaller than experimental work has shown it to be. The change in G is comparatively large, five units in the third figure. However it may very well be that there are as yet unsuspected sources of error, which if detected, will give a future experimental value of G compatible with our G. This was actually the situation in the experimental values of both e and h in the 1929 table. Our value of e is very close to the experimental value, a variation of two units in the fourth place. This is highly satisfactory because the experimental work for the determination of e has been extremely painstaking and is considered by Birge as an excellent value. As for Planck's constant the experimental value is too small by six units in the fourth place. We have no experimental value of λ as yet, but it is to be hoped that future observational work with the large telescopes now being prepared will provide such experimental data as to afford a comparison with our value.

These rather close checks between values derived from straight experimental work and those derived from an application of Eddington's theory to experimental results constitute an excellent recommendation for the theory.

The theory of Eddington's goes further than merely furnishing connections between constants of nature. It furnishes a badly needed link between relativity theory and quantum mechanics each of which has been built up independently and in such a way that for a time each seemed definitely to exclude the other. All efforts to graft quantum mechanics on to relativity theory failed, and Dirac succeeded in giving certain of his matrices tensor characteristics only by using methods which the relativists themselves had never thought of.

Eddington starts down in the sub-sub-basement and builds a structure which is capable of solving the hydrogen atom even to the detail of producing the fine structure of its spectral lines, then as we ascend through the edifice, we see clearly how relativity theory emerges from matrix theory. We even obtain improvements on the way to Weyl's beautiful derivation of Maxwell's equations in such a way as to eliminate one of his vexing contradictions, and by the time we reach the top of the structure we find ourselves passing judgment on the relative merits of the Einstein and the de Sitter models of the universe.

It is of course a bold theory that will announce the diameter of the universe with a degree of precision such that its probable error is only a few hundred light years. It is likewise a bold theory that will state the possible number of particles present in the universe with a precision of seventy-seven significant figures. But these very features will permit of a very clear-cut verification or an equally decisive destruction of the theory as soon as astronomical science catches up.

In the meantime it is no small triumph to have presented a theory which

was capable of showing that the 1929 values of Planck's constant and the electronic charge were too small by the type of percentage errors to be expected in a freshman physics laboratory. The subsequent revision of these two values from 6.547×10^{-27} to 6.624×10^{-27} erg-sec. and from 4.770×10^{-10} to 4.803×10^{-10} e.s.u. respectively is a startling corroboration. Again time will be the final judge.

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